SN – 251

III Semester B.A./B.Sc. Examination, Nov./Dec. 2014
(Semester Scheme) (O.S.)
(Prior to 2012-13)
MATHEMATICS – III

Time : 3 Hours   Max. Marks : 90

Instruction : *Answer **all** questions.*

I. Answer **any fifteen** questions : **(15×2=30)**

1) If H and K are any two subsets of G, then prove that $(HK)^{-1} = K^{-1} H^{-1}$.

2) Prove that the order of any power of an element of a group cannot exceed the order of the element.

3) Find the number of generators of the cyclic group $(Z_{18}, +_{18})$.

4) Let G be a cyclic group of order K and 'a' be a generator. If $a^m = a^n$ $(m \neq n)$, prove that $m \equiv n \pmod{k}$.

5) Prove that the index of any subgroup of a finite group is a divisor of the order of the group.

6) State Euler's theorem.

7) Define a convergent sequence.

8) Define supremum and infimum of a sequence.

9) Discuss the nature of the sequence $\{1 + (-1)^n\}$.

10) Define Harmonic series and write the nature of Harmonic series.

11) State Cauchy's root test for a series of positive terms.

12) If a series $\sum u_n$ is convergent, prove that $\lim\limits_{n \to \infty} u_n = 0$.

13) Test the convergence of the series $\sum \frac{n(-1)^{n-1}}{2n-1}$.

14) Sum to infinity the series $1 + 2\left(\frac{1}{9}\right) + \frac{2.5}{1.2}\left(\frac{1}{81}\right) + \frac{2.5.8}{1.2.3}\left(\frac{1}{729}\right) + \dots$

15) Examine the differentiability of the function $f(x) = \begin{cases} x^2 \sin\frac{1}{x}, & \text{when } x \neq 0 \\ 0, & \text{when } x = 0 \end{cases}$.

16) Verify Lagrange's mean value theorem for $f(x) = x(x-1)(x-2)$ in $\left[0, \frac{1}{2}\right]$.

17) Show that $\log(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} \ldots$ using Maclaurin's expansion.

18) Evaluate $\lim_{x \to 0} [x(\log x)]$.

19) Find the Fourier constant a_0 for $f(x) = e^x$ in $(-1, 1)$.

20) Find the half range sine series for $f(x) = x$ in $(0, \pi)$.

II. Answer **any three** questions : **(3×5=15)**

1) In a group G prove that $0(a) = 0(a^{-1})$ for every $a \in G$.
2) Let G be a cyclic group of order 'd' and 'a' be a generator. Prove that the element a^k $(k < d)$ is also a generator of G if and only if $(k, d) = 1$.
3) If H is a subgroup of a group G then prove that there exists a one-to-one correspondence between any two right cosets of H in G.
4) If 'a' is any integer and p is any positive prime then prove that $a^p \equiv a \pmod{p}$.
5) Prove that a finite group of prime order is cyclic and hence abelian.

III. Answer **any two** questions : **(2×5=10)**

1) Prove that every convergent sequence is bounded.
2) Discuss the nature of the sequence $\{x^n\}$ where x is real.
3) Test the convergence of the sequences

i) $\dfrac{\log(n+1) - \log n}{\sin\left(\frac{1}{n}\right)}$

ii) $\left(1 + \frac{a}{n}\right)^{n/b}$

IV. Answer **any three** questions : **(3×5=15)**

1) State and prove Leibnitz's test on alternating series.

2) Discuss the convergence of the series $\frac{5}{2+5}+\frac{5^2}{2^2+5}+\frac{5^3}{2^3+5}+\ldots$

3) Test the convergence of the series $\frac{x}{2\sqrt{3}}+\frac{x^2}{3\sqrt{4}}+\frac{x^3}{4\sqrt{5}}+\ldots$

4) Discuss the convergence of the series $\sum\frac{4.7.10\ldots(3n+1)x^n}{1.2.3.\ldots n}$

5) Sum to infinity, the series $\frac{2}{2!}\left(\frac{1}{12}\right)^2+\frac{2.5}{3!}\left(\frac{1}{12}\right)^3+\frac{2.5.8}{4!}\left(\frac{1}{12}\right)^4+\ldots$

V. Answer **any two** questions : **(2×5=10)**

1) State and prove Rolle's theorem.

2) Evaluate $\lim_{x\to 0}(\cos x)^{1/x^2}$.

3) Obtain the Maclaurin's expansion of $\log(1+\cos x)$ upto the term containing x^4.

4) Verify Cauchy's mean value theorem for $f(x)=e^x$, $g(x)=e^{-x}$ in (a, b).

VI. Answer **any two** questions : **(2×5=10)**

1) Find the Fourier series of $f(x)=e^{-x}$ in $(0, 2\pi)$.

2) Obtain the Fourier series of $f(x)=x^2$ in $(-\pi, \pi)$ and hence deduce that

$$\frac{\pi^2}{6}=1+\frac{1}{2^2}+\frac{1}{3^2}+\ldots$$

3) Obtain the half range cosine series for the function $f(x)=(x-1)^2$ in $(0, 1)$.
